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Note on Integrability of Marginally Deformed ABJ(M) Theories

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Abstract: We study the anomalous dimensions of operators in the scalar sector of β-deformed ABJ(M) theories. We show that the anomalous dimension matrix at two-loop order gives an integrable Hamiltonian acting on an alternating $SU(4)$ spin chain with the spins at odd lattice sides in the fundamental representation and the spins at even lattices in the anti-fundamental representation. We get a set of β-deformed Bethe ansatz equations which give the eigenvalues of Hamiltonian of this deformed spin chain system. Based on our computations, we also extend our study to non-supersymmetric three-parameter γ-deformation of ABJ(M) theories and find that the corresponding Hamiltonian is the same as the one in β-deformed case at two-loop level in the scalar sector.

Keywords: Gauge-gravity correspondence, Bethe ansatz, Supersymmetric gauge theory

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Contents

1 Introduction

There were great achievements on integrable structure in both sides of AdS/CFT correspondence in the last decade [1]. The best studied case is the integrability in the correspondence between four-dimensional $\mathcal{N} = 4$ super-Yang-Mills theory and the type IIB superstring theory on $AdS_5 \times S^5$ [2–4]. In the field theory side, it was found that, first at lower loop level, the anomalous dimension matrix coincides with Hamiltonian of certain integrable spin chain [5–7]. The all-loop Bethe ansatz equations [8] were later proposed and they can be obtained from a S-matrix for the spin chain [9], though we do not know the corresponding Hamiltonian of the spin chain at all-loop level. In the string theory side, infinity number of conserved charges on the worldsheet of Green-Schwarz superstring moving in $AdS_5 \times S^5$ were constructed [10]. The integrable structure is a very powerful tool which enables us to compute, as an example, the cusp anomalous dimension for arbitrary value of the 't Hooft coupling in the planar limit [11, 12].

Such integrable structure was also found for the recently proposed duality [13, 14] between three-dimensional $\mathcal{N} = 6$ Chern-Simons-matter theory and type IIA theory on $AdS_4 \times CP^3$ [15]-[42] (for reviews see [43, 44]). The dynamics in this case is more complicated and richer than the previous one, partly due to the fact that we now have less supersymmetries. As an example, there is still a to-be-determined function in the dispersion relation of the magnon [24, 25, 45]. In the $\mathcal{N} = 4$ super Yang-Mills case, this function

is trivial due to the fact that the theory is self-dual under the S-duality transformation [46].

With the success of integrability in mind, it should be with great value to generalize the above studies to case with less supersymmetries. The theory in which the anomalous dimensions of gauge invariant operators are related to an integrable Hamiltonian seems to be quite rare. If we perform generic marginal deformations of the $\mathcal{N} = 4$ super Yang-Mills theory [47] or ABJM theory, the obtained theory seems usually not to keep these integrable structure in the above sense, even when some supersymmetries and conformal symmetry are preserved. The β- and γ-deformations of $\mathcal{N} = 4$ super Yang-Mills theory are quite special since they preserve this remarkable integrable structure [48]-[51] and their gravity dual can be obtained through a certain solution generating technique [52]. Further studies of this integrability can be found in [53]-[73]. These marginal deformations are special also because they can be expressed elegantly using a star product which produces a certain phase factor for each interaction term in the Lagrangian. The solution generating transformation in the gravity side can be constructed by T-duality-shift-T-duality transformations in string theory [52]. The understand of the gauge-gravity correspondence in this case was improved in [74]. The β- and γ- deformed ABJM theories and their gravity duals were also studied in [74]. Some classical string solutions in these deformed backgrounds of type IIA string theory have been studied in [75]-[78]. The aim of this paper is to explore the integrable structure in the field theory side of these deformed AdS_4/CFT_3 correspondence.

We begin with the scalar sector of the β-deformed ABJ(M) theory which has $\mathcal{N} = 2$ supersymmetries. We compute the anomalous dimensions of these operators at the two-loop level in the planar limit and for all operators with length larger than 2. We express the result as a Hamiltonian of an alternating $SU(4)$ spin chain. Comparing with the undeformed case [15–17], we find that, in the Hamiltonian, only the terms from the interaction terms with six scalars are deformed. Though the interaction terms with two scalars and two fermions are also deformed, their contributions to the Hamiltonian coincide with the undeformed case. We also find that the Hamiltonian in non-supersymmetric three-parameter γ-deformed ABJ(M) theory is the same as the one of the β-deformed theory, at two-loop level in the scalar sector. We expect the differences will appear in other sectors and/or at higher loop order. As in [50], we deform the R-matrices constructed in [15, 16] by introducing suitable phase factors. We show that the obtained transfer matrices will produce essentially the Hamiltonian from the perturbative computations in the Chern-Simons-matter theories. This result shows that the Hamiltonian is integrable. By diagonalizing the transfer matrices, we obtain the Bethe ansatz equations and the eigenvalues of the Hamiltonian.

The organization of the remaining parts of this paper is as follows. In section 2 we briefly review β-deformation of ABJM theory. In section 3, we compute the two-loop corrections to the anomalous dimensions of operators in the scalar sector in both ABJM and ABJ theories. In section 4, we constructed the R matrices after deformation and show that the Hamiltonian obtained in section 2 is integrable. Based on these results, we derive the eigenvalues of the Hamiltonian of the deformed spin chain system in section 5. A brief discussion on non-supersymmetric three-parameter γ-deformation is put in

section 6. Section 7 is devoted to conclusions and discussions. We put some details of the computations in section 4 in the Appendix.

2 β-deformation of superconformal Chern-Simons-matter theory

The ABJM theory [13] is three dimensional $\mathcal{N}=6$ supersymmetric Chern-Simons-matter theory. The gauge group of this theory is $U(N)\times U(N)$, and the Chern-Simons levels of these two subgroups are k and $-k$, respectively. The matter fields are four complex scalars $Y^I, I=1,2,3,4$ and four fermions $\Psi_I, I=1,2,3,4$ in the $(N,\bar{N})$ representation. The action of this theory is *

$$S=\int d^3x(L_{CS}+L_{kin.}-V_F-V_B), \tag{2.1}$$

with

$$L_{CS}=\frac{k}{4\pi}\epsilon^{\mu\nu\rho}\mathrm{Tr}(A_\mu\partial_\mu A_\rho+\frac{2i}{3}A_\mu A_\nu A_\rho)-\frac{k}{4\pi}\epsilon^{\mu\nu\rho}\overline{\mathrm{Tr}}(\bar{A}_\mu\partial_\mu\bar{A}_\rho+\frac{2i}{3}\bar{A}_\mu\bar{A}_\nu\bar{A}_\rho), \tag{2.2}$$

$$\begin{aligned}L_{kin.}=&\frac{1}{2}\overline{\mathrm{Tr}}(-(D_\mu Y)^\dagger D^\mu Y^I+i\Psi^{\dagger I}\gamma_\mu D^\mu\Psi_I)+\frac{1}{2}\mathrm{Tr}(-D_\mu Y^I(D^\mu Y)^\dagger_I\\&+i\Psi_I\gamma_\mu D^\mu\Psi^{\dagger I}),\end{aligned} \tag{2.3}$$

$$\begin{aligned}V_{\mathrm{B}}=&-\frac{1}{3}\left(\frac{2\pi}{k}\right)^2\overline{\mathrm{Tr}}\Big[Y^\dagger_I Y^J Y^\dagger_J Y^K Y^\dagger_K Y^I+Y^\dagger_I Y^I Y^\dagger_J Y^J Y^\dagger_K Y^K\\&+4Y^\dagger_I Y^J Y^\dagger_K Y^I Y^\dagger_J Y^K-6Y^\dagger_I Y^I Y^\dagger_J Y^K Y^\dagger_K Y^J\Big],\end{aligned} \tag{2.4}$$

$$\begin{aligned}V_{\mathrm{F}}=&\frac{2\pi i}{k}\overline{\mathrm{Tr}}\Big[Y^\dagger_I Y^I\Psi^{\dagger J}\Psi_J-2Y^\dagger_I Y^J\Psi^{\dagger I}\Psi_J+\epsilon^{IJKL}Y^\dagger_I\Psi_J Y^\dagger_K\Psi_L]\\&-\frac{2\pi i}{k}\mathrm{Tr}[Y^I Y^\dagger_I\Psi_J\Psi^{\dagger J}-2Y^I Y^\dagger_J\Psi_I\Psi^{\dagger J}+\epsilon_{IJKL}Y^I\Psi^{\dagger J}Y^K\Psi^{\dagger L}\Big].\end{aligned} \tag{2.5}$$

The covariant derivatives are:

$$D_\mu Y^I=\partial_\mu Y^I+iA_\mu Y^I-iY^I\bar{A}_\mu, D_\mu Y^\dagger_I=\partial_\mu Y^\dagger_I+i\bar{A}_\mu Y^\dagger_I-iY^\dagger_I\bar{A}_\mu, \tag{2.6}$$

$$D_\mu\Psi_I=\partial_\mu\Psi_I+iA_\mu\Psi_I-i\Psi_I\bar{A}_\mu, D_\mu\Psi^{\dagger I}=\partial_\mu\Psi^{\dagger I}+i\bar{A}_\mu\Psi^{\dagger I}-i\Psi^{\dagger I}\bar{A}_\mu. \tag{2.7}$$

In the following part, we will discuss the β deformation of the theory following the convention given by [74]. The deformed theory will preserve three dimensional $\mathcal{N}=2$ supersymmetries. The β-deformation can be performed by replacing all of the ordinary product fg of two fields f and g in the Lagrangian by the following star product:

$$f*g=e^{i\pi\gamma(Q^f_1Q^g_2-Q^f_2Q^g_1)}fg\,, \tag{2.8}$$

*We follow the convention of [16] closely.

Table 1. $U(1)^2$ charges of the scalars of the ABJM theory used for β-deformation.

	Y^1	Y^2	Y^3	Y^4
$U(1)_1$	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0
$U(1)_2$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$

where $Q_i^f, i = 1, 2$ are two global $U(1)$ charges carrying by the field f and γ is a real deformation parameter. † For the β-deformation of ABJM theory, we choose the charges for the scalars as in table 1. The fermionic super-partner $\psi^{\dagger I}$ carries the same charge as Y^I, and the gauge field is neutral under these symmetries.

It is easy to see that this star product is associative and for several fields $F_1, ..., F_n$, we have

$$F_1 * ... * F_n = e^{i\pi\gamma \sum_{i<j}(Q_1^{F_i}Q_2^{F_j} - Q_2^{F_i}Q_1^{F_j})} F_1...F_n. \tag{2.9}$$

This deformation just adds phase factors according to the above equation to the interaction terms in the Lagrangian. One can see from the above rule for the star product that the deformation does not change the kinetic terms of the Lagrangian. In the superfield formulism, the deformation will only deform the superpotential W in the following manner given in [74]:

$$W \to W^{\text{deformed}} = \frac{4\pi}{k}\text{Tr}\left(e^{-i\pi\gamma/2}Y^1Y_3^\dagger Y^2Y_4^\dagger - e^{i\pi\gamma/2}Y^1Y_4^\dagger Y^2Y_3^\dagger\right), \tag{2.10}$$

where one can go back to the original quartic superpotential by setting deformation parameter γ to be zero.

For later use, we now give the interaction terms after the deformation. One can see that only V_B and V_F will be deformed. By a bit calculations, we found that the third term in scalar potential, eq. (2.4), will be deformed by multiplying the phase factor $\exp\left(-2i\pi\gamma(Q^I \times Q^J + Q^I \times Q^K + Q^K \times Q^I)\right)$. Here I, J, K take value of the integral number $1, 2, 3, 4$, and we define:

$$Q^I \times Q^J \equiv Q_1^I Q_2^J - Q_1^J Q_2^I. \tag{2.11}$$

Where Q^I are the $U(1)$ charges of the fields.

†We denote the deformation parameter as γ to stress that it is *real*, However the supersymmetric one-parameter deformation is still called β-deformation and the non-supersymmetric three-parameter deformation to be introduced in section 6 will be called γ-deformation. We hope this will not produce confusions for the readers.

Table 2. The non-vanishing phase factors for the third term of V_B

I	J	K	phase factor
1	2	3	$i\pi\gamma$
1	2	4	$-i\pi\gamma$
1	3	2	$-i\pi\gamma$
1	3	4	$-i\pi\gamma$
1	4	2	$i\pi\gamma$
1	4	3	$i\pi\gamma$

Table 3. The non-vanishing phase factors for the second term of V_F

I	J	phase factor
1	3	$-\frac{1}{2}i\pi\gamma$
1	4	$\frac{1}{2}i\pi\gamma$
2	3	$\frac{1}{2}i\pi\gamma$
2	4	$-\frac{1}{2}i\pi\gamma$
3	1	$\frac{1}{2}i\pi\gamma$
3	2	$-\frac{1}{2}i\pi\gamma$
4	1	$-\frac{1}{2}i\pi\gamma$
4	2	$\frac{1}{2}i\pi\gamma$

So the third term now becomes:

$$
\begin{aligned}
&V^{\text{deformed}}_{\text{B, 3rd}} \\
&= -\frac{1}{3}\left(\frac{2\pi}{k}\right)^2 \Bigg[4 \sum_{\text{two of I, J, K are the same}} \overline{\mathrm{Tr}}(Y_I^+ Y^J Y_k^+ Y^I Y_J^+ Y^K) \\
&+ 4 \sum_{\text{all of I, J, K belong to (12) or (34)}} \overline{\mathrm{Tr}}(Y_I^+ Y^J Y_k^+ Y^I Y_J^+ Y^K) \\
&+ 4 \sum_{(IJK)=(123),(143),(142)} e^{i\pi\gamma}\overline{\mathrm{Tr}}(Y_I^+ Y^J Y_k^+ Y^I Y_J^+ Y^K) + \text{cyclic permutations} \\
&+ 4 \sum_{(IJK)=(132),(134),(124)} e^{-i\pi\gamma}\overline{\mathrm{Tr}}(Y_I^+ Y^J Y_k^+ Y^I Y_J^+ Y^K) + \text{cyclic permutations} \Bigg] . \qquad (2.12)
\end{aligned}
$$

The non-vanishing phase factors in the third term of the potential V_B are listed in table 2. The other three terms in eq. (2.4) are untouched by the β-deformation.

Now we turn to consider the interaction terms between the fermions and scalars in

Table 4. The non-vanishing phase factors for the third term of V_F

I	J	K	L	phase factor
1	3	2	4	$\frac{1}{2}i\pi\gamma$
1	4	2	3	$-\frac{1}{2}i\pi\gamma$
2	3	1	4	$-\frac{1}{2}i\pi\gamma$
2	4	1	3	$\frac{1}{2}i\pi\gamma$
3	1	4	2	$-\frac{1}{2}i\pi\gamma$
3	2	4	1	$\frac{1}{2}i\pi\gamma$
4	1	3	2	$\frac{1}{2}i\pi\gamma$
4	2	3	1	$-\frac{1}{2}i\pi\gamma$

eq. (2.5). After the deformation, these terms become

$$\begin{aligned} V_F^{\text{deformed}} = \frac{2\pi i}{k}\mathrm{Tr}\Big[& Y_I^+ Y^I \Psi^{+J}\Psi_J - 2 \sum_{I=J,\text{ or } I,J\in\{1,2\},\text{ or } I,J\in\{3,4\}} Y_I^+ Y^J \Psi^{+I}\Psi_J \\ & -2 \sum_{(I,J)=(1,4),(2,3),(3,1),(4,2)} e^{\frac{i}{2}\pi\gamma} Y_I^+ Y^J \Psi^{+I}\Psi_J \\ & -2 \sum_{(I,J)=(1,3),(2,4),(3,2),(4,1)} e^{-\frac{i}{2}\pi\gamma} Y_I^+ Y^J \Psi^{+I}\Psi_J \\ & + \sum_{(IJKL)=(1324),(2413),(3241),(4132)} e^{\frac{i}{2}\pi i\gamma} Y_I^+ \Psi_J Y_K^+ \Psi^L \\ & + \sum_{(IJKL)=(1423),(2314),(3142),(4231)} e^{-\frac{i}{2}\pi i\gamma} Y_I^+ \Psi_J Y_K^+ \Psi_L \\ & + \sum_{\text{other terms}} \epsilon^{IJKL} \Psi_I^+ \Psi_J Y_K^+ \Psi_L \Big] + \text{h.c} \end{aligned} \tag{2.13}$$

We list the non-vanishing phase factors multiplying the second and the third terms of V_F get changed in tables 3 and 4.

3 Two-loop anomalous dimensions in the scalar sector

Now we compute the two-loop planar contributions to the anomalous dimensions for the composite local operators in the scalar sector:

$$\mathcal{O}^{I_1\cdots I_L}_{J_1\cdots J_L} \equiv \mathrm{Tr}\left(Y^{I_1}Y^\dagger_{J_1}Y^{I_2}Y^\dagger_{J_2}\cdots Y^{I_L}Y^\dagger_{J_L}\right),\ L\geq 2. \tag{3.1}$$

The anomalous dimension matrix can be expressed as Hamiltonian acting on an alternating $SU(4)$ spin chain with the spins at odd lattice sides in the fundamental representation ($\mathbf{4}$) and the spins at even lattices in the anti-fundamental representation ($\bar{\mathbf{4}}$). The length of the spin chain is $2L$. The involved Feynman diagrams are the same as the ones in [15, 16].

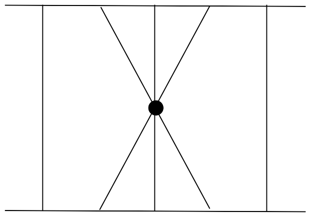

Figure 1. The contribution to anomalous dimension of $\mathcal{O}$ from two loop contribution from scalar sextet interaction. In this context, the horizontal lines represent the operators and the ordered vertical lines denote the contraction between the two operators of the fields included in trace.

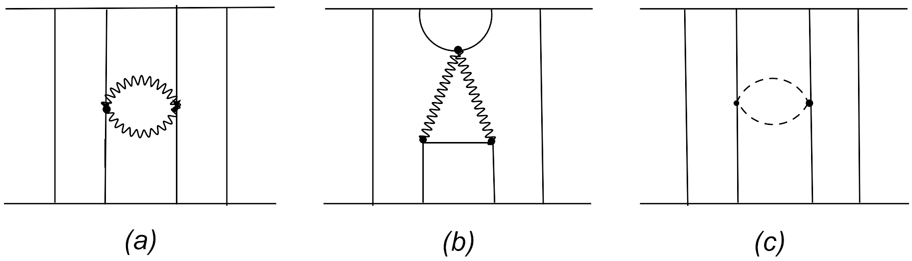

Figure 2. The contribution to anomalous dimension of $\mathcal{O}$ from two loop contribution of gauge and fermion exchange interaction. The internal waved lines and dashed line stand gluon and scalar respectively.

Since only the third terms in V_B is modified by the deformation, one can see by some computations based on the Feynman diagram in Fig. 1 that the Hamiltonian from V_B,

$$H_B = \frac{\lambda^2}{2}\sum_{i=1}^{2L}[\mathbb{I} - 2\mathbb{P}_{i,i+2} - \mathbb{K}_{i,i+1} + \mathbb{P}_{i,i+2}\mathbb{K}_{i,i+1} + \mathbb{K}_{i,i+1}\mathbb{P}_{i,i+1}]\,, \tag{3.2}$$

is now changed into

$$\widetilde{H}_B = \frac{\lambda^2}{2}\sum_{i=1}^{2L}[\mathbb{I} - 2\widetilde{\mathbb{P}}_{i,i+2} - \mathbb{K}_{i,i+1} + \mathbb{P}_{i,i+2}\mathbb{K}_{i,i+1} + \mathbb{K}_{i,i+1}\mathbb{P}_{i,i+1}]\,, \tag{3.3}$$

where $\lambda \equiv N/k$ is the 't Hooft coupling of ABJM theory, and definition of $\mathbb{I}, \mathbb{P}, \mathbb{K}$ are

$$\mathbb{I}^{IJ}_{KL} = \delta^I_K\delta^J_L,\ \mathbb{P}^{IJ}_{KL} = \delta^I_L\delta^J_K,\ \mathbb{K}^{IJ}_{KL} = \delta^{IJ}\delta_{KL}. \tag{3.4}$$

The definition of $\widetilde{\mathbb{P}}_{i,i+2}$ is

$$\begin{aligned}\left(\widetilde{\mathbb{P}}_{i,i+2}\right)^{I_iI_{i+1}I_{i+2}}_{J_iJ_{i+1}J_{i+2}} &\equiv \exp(-i\pi\gamma(Q^{J_i}\times Q^{J_{i+1}} + Q^{J_{i+1}}\times Q^{J_{i+2}} + Q^{J_{i+2}}\times Q^{J_i}))\\ &\quad\times\ (\mathbb{P}_{i,i+2})^{I_iI_{i+1}I_{i+2}}_{J_iJ_{i+1}J_{i+2}}\,.\end{aligned} \tag{3.5}$$

The contributions to the anomalous dimension of operator (3.1) from gauge and fermion exchange interaction are relevant to Feynman diagrams in Fig. 2. The wave

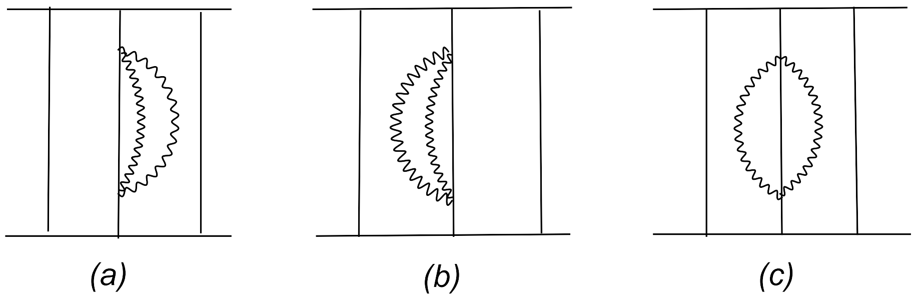

Figure 3. The contribution to wave function renormalization of Y, Y^+ from two loop contribution of diamagnetic gauge interactions.

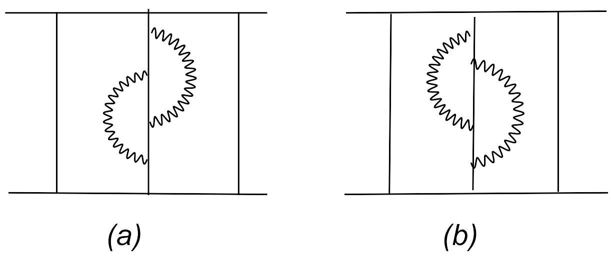

Figure 4. The contribution to wave function renormalization of Y, Y^+ is from two loop contribution of paramagnetic gauge interactions.

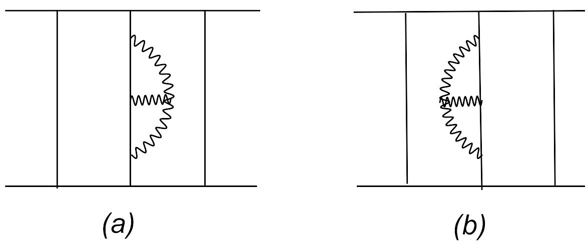

Figure 5. The contribution to wave function renormalization of Y, Y^+ is from two loop contribution of Chern-Simons interaction.

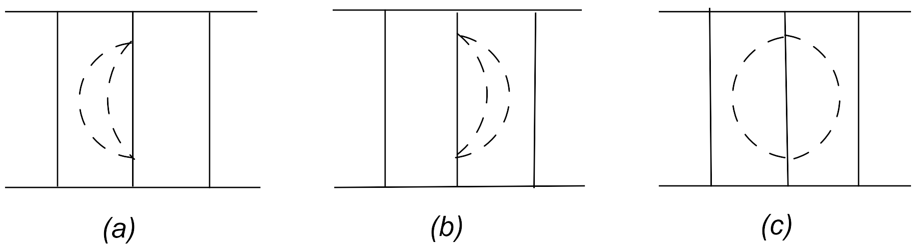

Figure 6. The contribution to wave function renormalization of Y, Y^+ is from two loop contribution of fermion pair interaction to wave function renormalization.

function renormalization of Y, Y^+ will also make contributions to anomalous dimension of composite operators in eq. (3.1). There are three kinds of nonzero contribution arise from interactions involving gauge boson loops, vertices given in V_F^{deformed} and gauge-matter

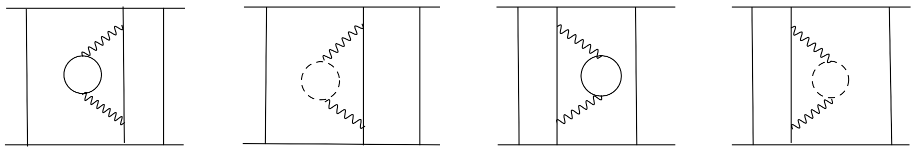

Figure 7. The contribution to wave function renormalization of Y, Y^+ is from two loop contribution of vacuum polarization.

interactions. The relevant Feynman diagrams are given in Fig. 3-Fig. 5, Fig. 6 and Fig. 7, respectively. We find that these contributions

$$H_F = \lambda^2 \sum_{i=1}^{2L} \mathbb{K}_{i,i+1}, \tag{3.6}$$

$$H_{\text{gauge}} = \lambda^2 \sum_{i=1}^{2L} \left(-\frac{1}{4}\mathbb{I} - \frac{1}{2}\mathbb{K}_{i,i+1} \right), \tag{3.7}$$

$$H_Z = \lambda^2 \sum_{i=1}^{2L} \frac{3}{4}\mathbb{I}, \tag{3.8}$$

are the same as the undeformed case.

By summing over all of these contributions, we get [‡]

$$\begin{aligned} \widetilde{H}_{\text{total}} &= \widetilde{H}_B + H_F + H_{gauge} + H_Z \\ &= \lambda^2 \sum_{i=1}^{2L} \left(\mathbb{I} - \widetilde{\mathbb{P}}_{i,i+2} + \frac{1}{2}\mathbb{P}_{i,i+2}\mathbb{K}_{i,i+1} + \frac{1}{2}\mathbb{P}_{i,i+2}\mathbb{K}_{i+1,i+2} \right). \end{aligned} \tag{3.10}$$

We notice that, as the undeformed case [17], the computations in the ABJ theory [14] with gauge group $U(N)_k \times U(M)_{-k}$ is almost the same besides replacing the factor λ^2 in the Hamiltonian by the factor $\lambda\tilde{\lambda}$, where $\tilde{\lambda}$ is defined to be $\tilde{\lambda} \equiv M/k$. So the computations and discussions in the following applied to the scalar section in ABJ theory at two-loop level as well.

4 The R matrices of deformed spin chain

In this section, we will show that Hamiltonian obtained in the previous section is integrable by constructing the R matrix which satisfies the Yang-Baxter equation (YBE) and gives this Hamiltonian through the transfer matrix by the standard procedure.

‡ Here our convention is

$$\begin{aligned} &(\mathbb{P}_{i,i+2}\mathbb{K}_{i+1,i+2})^{K_i K_{i+1} K_{i+2}}_{J_i J_{i+1} J_{i+2}} \\ =\ &(\mathbb{P}_{i,i+2})^{K_i K_{i+1} K_{i+2}}_{L_i L_{i+1} L_{i+2}} (\mathbb{K}_{i+1,i+2})^{L_i L_{i+1} L_{i+2}}_{J_i J_{i+1} J_{i+2}} \end{aligned} \tag{3.9}$$

For the alternating spin chain, we need four R-matrices $\widetilde{\mathfrak{R}^{\mathbf{44}}_{ij}}(u), \widetilde{\mathfrak{R}^{\bar{\mathbf{4}}\mathbf{4}}_{ij}}(u), \widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}_{ij}}(u), \widetilde{\mathfrak{R}^{\bar{\mathbf{4}}\bar{\mathbf{4}}}_{ij}}(u)$ acting on the space $V_i \otimes V_j$. Here the upper indices of $\widetilde{\mathfrak{R}^{\mathbf{44}}_{ij}}(u)$ denote $SU(4)$ representations related to the two spaces and u denotes spectral parameter. We defined these R-matrices as

$$\begin{aligned}
\widetilde{\mathfrak{R}^{\mathbf{44}}}(u)^{IJ}_{KL} &= \exp(-\frac{i}{2}\pi\gamma(Q^J \times Q^I - Q^K \times Q^L))\mathfrak{R}^{\mathbf{44}}(u)^{IJ}_{KL}, && (4.1)\\
\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}(u)^{IJ}_{KL} &= \exp(\frac{i}{2}\pi\gamma(Q^J \times Q^I - Q^K \times Q^L))\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}(u)^{IJ}_{KL}, && (4.2)\\
\widetilde{\mathfrak{R}^{\bar{\mathbf{4}}\mathbf{4}}}(u)^{IJ}_{KL} &= \exp(\frac{i}{2}\pi\gamma(Q^J \times Q^I - Q^K \times Q^L))\mathfrak{R}^{\bar{\mathbf{4}}\mathbf{4}}(u)^{IJ}_{KL}, && (4.3)\\
\widetilde{\mathfrak{R}^{\bar{\mathbf{4}}\bar{\mathbf{4}}}}(u)^{IJ}_{KL} &= \exp(-\frac{i}{2}\pi\gamma(Q^J \times Q^I - Q^K \times Q^L))\mathfrak{R}^{\bar{\mathbf{4}}\bar{\mathbf{4}}}(u)^{IJ}_{KL}, && (4.4)
\end{aligned}$$

where

$$\begin{aligned}
\mathfrak{R}^{\mathbf{44}}(u) &= u\mathbb{I} + \mathbb{P}, && (4.5)\\
\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}(u) &= -(u+2)\mathbb{I} + \mathbb{K}, && (4.6)\\
\mathfrak{R}^{\bar{\mathbf{4}}\mathbf{4}}(u) &= -(u+2)\mathbb{I} + \mathbb{K}, && (4.7)\\
\mathfrak{R}^{\bar{\mathbf{4}}\bar{\mathbf{4}}}(u) &= u\mathbb{I} + \mathbb{P}, && (4.8)
\end{aligned}$$

are the R-matrices beform deformation [15, 16]. From above formulas, we can get:

$$\begin{aligned}
\widetilde{\mathfrak{R}^{\mathbf{44}}}(u)^{IJ}_{KL} &= u\exp(-i\pi\gamma Q^J \times Q^I)\delta^I_K\delta^J_L + \delta^I_L\delta^J_K, && (4.9)\\
\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}(u)^{IJ}_{KL} &= -(u+2)\exp(i\pi\gamma Q^J \times Q^I)\delta^I_K\delta^J_L + \delta^{IJ}\delta_{KL}, && (4.10)\\
\widetilde{\mathfrak{R}^{\bar{\mathbf{4}}\mathbf{4}}}(u)^{IJ}_{KL} &= -(u+2)\exp(i\pi\gamma Q^J \times Q^I)\delta^I_K\delta^J_L + \delta^{IJ}\delta_{KL}, && (4.11)\\
\widetilde{\mathfrak{R}^{\bar{\mathbf{4}}\bar{\mathbf{4}}}}(u)^{IJ}_{KL} &= u\exp(-i\pi\gamma Q^J \times Q^I)\delta^I_K\delta^J_L + \delta^I_L\delta^J_K. && (4.12)
\end{aligned}$$

The R matrices before deformation satisfy YBE [15, 16]:

$$\begin{aligned}
\mathfrak{R}^{\mathbf{44}}{}_{12}(u-v)\mathfrak{R}^{\mathbf{44}}{}_{13}(u)\mathfrak{R}^{\mathbf{44}}{}_{23}(v) &= \mathfrak{R}^{\mathbf{44}}{}_{23}(v)\mathfrak{R}^{\mathbf{44}}{}_{13}(u)\mathfrak{R}^{\mathbf{44}}{}_{12}(u-v), && (4.13)\\
\mathfrak{R}^{\mathbf{44}}{}_{12}(u-v)\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}{}_{13}(u)\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}{}_{23}(v) &= \mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}{}_{23}(v)\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}{}_{13}(u)\mathfrak{R}^{\mathbf{44}}{}_{12}(u-v), && (4.14)\\
\mathfrak{R}^{\bar{\mathbf{4}}\bar{\mathbf{4}}}{}_{12}(u-v)\mathfrak{R}^{\bar{\mathbf{4}}\mathbf{4}}{}_{13}(u)\mathfrak{R}^{\bar{\mathbf{4}}\mathbf{4}}{}_{23}(v) &= \mathfrak{R}^{\bar{\mathbf{4}}\mathbf{4}}{}_{23}(v)\mathfrak{R}^{\bar{\mathbf{4}}\mathbf{4}}{}_{13}(u)\mathfrak{R}^{\bar{\mathbf{4}}\bar{\mathbf{4}}}{}_{12}(u-v), && (4.15)\\
\mathfrak{R}^{\bar{\mathbf{4}}\bar{\mathbf{4}}}{}_{12}(u-v)\mathfrak{R}^{\bar{\mathbf{4}}\bar{\mathbf{4}}}{}_{13}(u)\mathfrak{R}^{\bar{\mathbf{4}}\bar{\mathbf{4}}}{}_{23}(v) &= \mathfrak{R}^{\bar{\mathbf{4}}\bar{\mathbf{4}}}{}_{23}(v)\mathfrak{R}^{\bar{\mathbf{4}}\bar{\mathbf{4}}}{}_{13}(u)\mathfrak{R}^{\bar{\mathbf{4}}\bar{\mathbf{4}}}{}_{12}(u-v). && (4.16)
\end{aligned}$$

As in [50], the choice of the phases in eqs. (4.1-4.4) are such that the R matrices after deformation still satisfy the YBE:

$$\begin{aligned}
\widetilde{\mathfrak{R}^{\mathbf{44}}}{}_{12}(u-v)\widetilde{\mathfrak{R}^{\mathbf{44}}}{}_{13}(u)\widetilde{\mathfrak{R}^{\mathbf{44}}}{}_{23}(v) &= \widetilde{\mathfrak{R}^{\mathbf{44}}}{}_{23}(v)\widetilde{\mathfrak{R}^{\mathbf{44}}}{}_{13}(u)\widetilde{\mathfrak{R}^{\mathbf{44}}}{}_{12}(u-v), && (4.17)\\
\widetilde{\mathfrak{R}^{\mathbf{44}}}{}_{12}(u-v)\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}{}_{13}(u)\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}{}_{23}(v) &= \widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}{}_{23}(v)\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}{}_{13}(u)\widetilde{\mathfrak{R}^{\mathbf{44}}}{}_{12}(u-v), && (4.18)\\
\widetilde{\mathfrak{R}^{\bar{\mathbf{4}}\bar{\mathbf{4}}}}{}_{12}(u-v)\widetilde{\mathfrak{R}^{\bar{\mathbf{4}}\mathbf{4}}}{}_{13}(u)\widetilde{\mathfrak{R}^{\bar{\mathbf{4}}\mathbf{4}}}{}_{23}(v) &= \widetilde{\mathfrak{R}^{\bar{\mathbf{4}}\mathbf{4}}}{}_{23}(v)\widetilde{\mathfrak{R}^{\bar{\mathbf{4}}\mathbf{4}}}{}_{13}(u)\widetilde{\mathfrak{R}^{\bar{\mathbf{4}}\bar{\mathbf{4}}}}{}_{12}(u-v), && (4.19)\\
\widetilde{\mathfrak{R}^{\bar{\mathbf{4}}\bar{\mathbf{4}}}}{}_{12}(u-v)\widetilde{\mathfrak{R}^{\bar{\mathbf{4}}\bar{\mathbf{4}}}}{}_{13}(u)\widetilde{\mathfrak{R}^{\bar{\mathbf{4}}\bar{\mathbf{4}}}}{}_{23}(v) &= \widetilde{\mathfrak{R}^{\bar{\mathbf{4}}\bar{\mathbf{4}}}}{}_{23}(v)\widetilde{\mathfrak{R}^{\bar{\mathbf{4}}\bar{\mathbf{4}}}}{}_{13}(u)\widetilde{\mathfrak{R}^{\bar{\mathbf{4}}\bar{\mathbf{4}}}}{}_{12}(u-v). && (4.20)
\end{aligned}$$

By introducing an auxiliary space V_0, we can define the following two transfer T-matrices [§]:

$$\widetilde{T}_0(u,a) = 2^{-L}\widetilde{\mathfrak{R}^{\mathbf{4}\mathbf{4}}}_{01}(u)\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}_{02}(u+a)\widetilde{\mathfrak{R}^{\mathbf{4}\mathbf{4}}}_{03}(u)\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}_{04}(u+a)\cdots \widetilde{\mathfrak{R}^{\mathbf{4}\mathbf{4}}}_{0(2L-1)}(u)\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}_{0(2L)}(u+a)\,, \tag{4.21}$$

$$\widetilde{\widetilde{T}}_0(u,\bar{a}) = 2^{-L}\widetilde{\mathfrak{R}^{\bar{\mathbf{4}}\mathbf{4}}}_{01}(u+\bar{a})\widetilde{\mathfrak{R}^{\bar{\mathbf{4}}\bar{\mathbf{4}}}}_{02}(u)\widetilde{\mathfrak{R}^{\bar{\mathbf{4}}\mathbf{4}}}_{03}(u+\bar{a})\widetilde{\mathfrak{R}^{\bar{\mathbf{4}}\bar{\mathbf{4}}}}_{04}(u)\cdots \widetilde{\mathfrak{R}^{\bar{\mathbf{4}}\mathbf{4}}}_{0(2L-1)}(u+\bar{a})\widetilde{\mathfrak{R}^{\bar{\mathbf{4}}\bar{\mathbf{4}}}}_{0(2L)}(u)\,. \tag{4.22}$$

The YBE will lead to the following relations:

$$\widetilde{\mathfrak{R}^{\mathbf{4}\mathbf{4}}}_{00'}(\nu-\mu)\widetilde{T}_0(\nu,a)\widetilde{T}_{0'}(\mu,a) = \widetilde{T}_{0'}(\mu,a)\widetilde{T}_0(\nu,a)\widetilde{\mathfrak{R}^{\mathbf{4}\mathbf{4}}}_{00'}(\nu-\mu), \tag{4.23}$$

$$\widetilde{\mathfrak{R}^{\bar{\mathbf{4}}\bar{\mathbf{4}}}}_{00'}(\nu-\mu)\widetilde{\widetilde{T}}_0(\nu,\bar{a})\widetilde{\widetilde{T}}_{0'}(\mu,\bar{a}) = \widetilde{\widetilde{T}}_{0'}(\mu,\bar{a})\widetilde{\widetilde{T}}_0(\nu,\bar{a})\widetilde{\mathfrak{R}^{\bar{\mathbf{4}}\bar{\mathbf{4}}}}_{00'}(\nu-\mu), \tag{4.24}$$

$$\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}_{00'}(\nu-\mu+a)\widetilde{T}_0(\nu,a)\widetilde{\widetilde{T}}_{0'}(\mu,-a) = \widetilde{\widetilde{T}}_{0'}(\mu,-a)\widetilde{T}_0(\nu,a)\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}_{00'}(\nu-\mu+a). \tag{4.25}$$

Then the traces of the two T-matrices

$$\widetilde{\tau}(\nu,a) = \operatorname*{Tr}_0 \widetilde{T}_0(\nu,a)\,, \tag{4.26}$$

$$\widetilde{\widetilde{\tau}}(\nu,a) = \operatorname*{Tr}_0 \widetilde{\widetilde{T}}_0(\nu,a)\,, \tag{4.27}$$

satisfy

$$\begin{aligned}&[\widetilde{\tau}(\nu,a),\widetilde{\tau}(\mu,a)] = 0\\ &[\widetilde{\widetilde{\tau}}(\nu,\bar{a}),\widetilde{\widetilde{\tau}}(\mu,\bar{a})] = 0\,,\\ &[\widetilde{\tau}(\nu,a),\widetilde{\widetilde{\tau}}(\mu,-a)] = 0\,.\end{aligned} \tag{4.28}$$

From now on, we restrict a to be purely imaginary so that $\widetilde{\tau}(\nu,a)$ and $\widetilde{\widetilde{\tau}}(\mu,\bar{a})$ commute with each other.

Now we compute the Hamiltonian from the R-matrices:

$$H = \frac{\partial \log \widetilde{\tau}(u,a)}{\partial u}\Big|_{u=0} + \frac{\partial \log \widetilde{\widetilde{\tau}}(u,\bar{a})}{\partial u}\Big|_{u=0}. \tag{4.29}$$

After some computations, we get

$$H = \sum_{i=1}^{2L} H_i, \tag{4.30}$$

with

$$\begin{aligned}H_{2l-1} &= \frac{1}{a^2-4}((a-2)\mathbb{I} + (a^2-4)\widetilde{\mathbb{P}}_{2l-1,2l+1}\\ &- (a-2)\mathbb{P}_{2l-1,2l+1}\mathbb{K}_{2l-1,2l} + (a+2)\mathbb{P}_{2l-1,2l+1}\mathbb{K}_{2l,2l+1})\,,\end{aligned} \tag{4.31}$$

$$\begin{aligned}H_{2l} &= \frac{1}{a^2-4}(-(a+2)\mathbb{I} + (a^2-4)\widetilde{\mathbb{P}}_{2l,2l+2}\\ &+ (a+2)\mathbb{P}_{2l,2l+2}\mathbb{K}_{2l,2l+1} - (a-2)\mathbb{P}_{2l,2l+2}\mathbb{K}_{2l+1,2l+2})\,.\end{aligned} \tag{4.32}$$

[§]Comparing with the T-matrices in [16], we include the constant factor 2^{-L} as in [15].

Here we have already used the relation $\bar{a} = -a$. The details of the computations are deferred to the Appendix. After setting $a = 0$, multiplying H_i by -1, then shifting H_i by $\frac{3}{2}\mathbb{I}$, the above Hamiltonian coincides with the one in the previous section obtained from the perturbative computations in field theory side.

5 Eigenvalues of deformed spin chain Hamiltonian and Bethe ansatz equations

In previous section, we constructed transfer matrix and Hamiltonian of the deformed spin chain. We now derive the Bethe ansatz equation through diagonalizing the transfer matrices. By choosing the ground state or highest-weight state as $|1\bar{4}1\bar{4}\cdots>$ and introducing three sets of Bethe roots $(l_a, m_b, r_c), 1 \leq a \leq N_l, 1 \leq b \leq N_m, 1 \leq c \leq N_r$, we can get the eigenvalues of $\tilde{\tau}(\nu, 0)$ as

$$\begin{aligned}\widetilde{\Lambda}(\nu) = & \, 2^{-L}(\nu+1)^L(-\nu-2)^L \exp(\frac{i}{4}\pi\gamma L + \frac{i}{4}\gamma N_m - \frac{i}{2}\pi\gamma N_r) \prod_{a=1}^{N_l} \frac{\nu + il_a - \frac{1}{2}}{\nu + il_a + \frac{1}{2}} \\ & + 2^{-L}(\nu+1)^L(-\nu)^L \exp(\frac{i}{4}\pi\gamma L + \frac{i}{4}\gamma N_m - \frac{i}{2}\pi\gamma N_l) \prod_{c=1}^{N_r} \frac{\nu + ir_c + \frac{5}{2}}{\nu + ir_c + \frac{3}{2}} \\ & + 2^{-L}(-\nu-2)^L\nu^L \exp(-\frac{i}{4}\pi\gamma L - \frac{i}{4}\pi\gamma N_m + \frac{i}{2}\pi\gamma N_r) \prod_{a=1}^{N_l} \frac{\nu + il_a + \frac{3}{2}}{\nu + il_a + \frac{1}{2}} \prod_{b=1}^{N_m} \frac{\nu + im_b}{\nu + im_b + 1} \\ & + 2^{-L}(-\nu-2)^L\nu^L \exp(-\frac{i}{4}\pi\gamma L - \frac{i}{4}\pi\gamma N_m + \frac{i}{2}\pi\gamma N_l) \prod_{c=1}^{N_c} \frac{\nu + ir_c + \frac{1}{2}}{\nu + ir_c + \frac{3}{2}} \prod_{b=1}^{N_m} \frac{\nu + im_b + 2}{\nu + im_b + 1}.\end{aligned} \tag{5.1}$$

As the undeformed case [15, 16], N_l, N_m, N_r should satisfy:

$$2N_l \leq L + N_m,\; 2N_r \leq L + N_m,\; 2N_m \leq N_l + N_r. \tag{5.2}$$

Similarly, we can get the eigenvalues of $\tilde{\tilde{\tau}}(\nu, 0)$ as

$$\begin{aligned}\widetilde{\widetilde{\Lambda}}(\nu) = & \, 2^{-L}(-\nu)^L(\nu+1)^L \exp(-\frac{i}{4}\pi\gamma L - \frac{i}{4}\pi\gamma N_m + \frac{i}{2}\pi\gamma N_r) \prod_{a=1}^{N_l} \frac{\nu + il_a + \frac{5}{2}}{\nu + il_a + \frac{3}{2}} \\ & + 2^{-L}(\nu+1)^L(-\nu-2)^L \exp(-\frac{i}{4}\pi\gamma L - \frac{i}{4}\pi\gamma N_m + \frac{i}{2}\pi\gamma N_l) \prod_{c=1}^{N_r} \frac{\nu + ir_c - \frac{1}{2}}{\nu + ir_c + \frac{1}{2}} \\ & + 2^{-L}(-\nu-2)^L\nu^L \exp(\frac{i}{4}\pi\gamma L + \frac{i}{4}\pi\gamma N_m - \frac{i}{2}\pi\gamma N_r) \prod_{a=1}^{N_l} \frac{\nu + il_a + \frac{1}{2}}{\nu + il_a + \frac{3}{2}} \prod_{b=1}^{N_m} \frac{\nu + im_b + 2}{\nu + im_b + 1} \\ & + 2^{-L}(-\nu-2)^L\nu^L \exp(\frac{i}{4}\pi\gamma L + \frac{i}{4}\pi\gamma N_m - \frac{i}{2}\pi\gamma N_l) \prod_{b=1}^{N_m} \frac{\nu + im_b}{\nu + im_b + 1} \prod_{c=1}^{N_r} \frac{\nu + ir_c + \frac{3}{2}}{\nu + ir_c + \frac{1}{2}}.\end{aligned} \tag{5.3}$$

By demanding the residue vanishes at every pole of $\widetilde{\Lambda}(\nu)$, we get the following set of Bethe Ansatz equations.

$$\begin{aligned}\exp(-\frac{i}{2}\pi\gamma L-\frac{i}{2}\pi\gamma N_m+i\pi\gamma N_l)\left(\frac{l_a-\frac{i}{2}}{l_a+\frac{i}{2}}\right)^L &= \prod_{a'\neq a}\frac{l_a-l_{a'}-i}{l_a-l_{a'}+i}\prod_{b=1}^{N_m}\frac{l_a-m_b+\frac{i}{2}}{l_a-m_b-\frac{i}{2}},\\ \exp(\frac{i}{2}\pi\gamma L+\frac{i}{2}\pi\gamma N_m-i\pi\gamma N_l)\left(\frac{r_c-\frac{i}{2}}{r_c+\frac{i}{2}}\right)^L &= \prod_{b=1}^{N_m}\frac{r_c-m_b+\frac{i}{2}}{r_c-m_b-\frac{i}{2}}\prod_{c'\neq c}\frac{r_c-r_{c'}-i}{r_c-r_{c'}+i},\\ \exp(\frac{i}{2}\pi\gamma N_l-\frac{i}{2}\pi\gamma N_r)=\prod_{a=1}^{N_l}\frac{m_b-l_a-\frac{i}{2}}{m_b-l_a+\frac{i}{2}}&\prod_{b\neq b'}\frac{m_b-m_{b'}-i}{m_b-m_{b'}+i}\prod_{c=1}^{N_r}\frac{m_b-r_c+\frac{i}{2}}{m_b-r_c-\frac{i}{2}}. \end{aligned} \tag{5.4}$$

We will get the same set of equations if we start with $\widetilde{\widetilde{\Lambda}}(\nu)$ instead. It is a consistent check that the same set of Bethe ansatz equations remove potential simple pole terms for $\widetilde{\Lambda}(\nu)$ and $\widetilde{\widetilde{\Lambda}}(\nu)$. One can see the equations just above go back to Bethe ansatz equations given in [15, 16] by setting $\gamma = 0$.

From the above eigenvalues, we can get the total momentum as

$$\begin{aligned} P_{total} &= \frac{1}{i}\left[\log\widetilde{\Lambda}(0)+\log\widetilde{\widetilde{\Lambda}}(0)\right]\\ &= \frac{1}{i}\left[-\frac{i}{2}\pi\gamma N_r+\frac{i}{2}\pi\gamma N_l+\sum_{a=1}^{N_l}\log\frac{il_a-\frac{1}{2}}{il_a+\frac{1}{2}}+\sum_{c=1}^{N_r}\log\frac{ir_c-\frac{1}{2}}{ir_c+\frac{1}{2}}\right]. \end{aligned} \tag{5.5}$$

From the vanishing of the total momentum, we can obtain the following constraint:

$$1=\exp(-\frac{i}{2}\pi\gamma N_r+\frac{i}{2}\pi\gamma N_l)\prod_{a=1}^{N_l}\frac{il_a-\frac{1}{2}}{il_a+\frac{1}{2}}\prod_{c=1}^{N_r}\frac{ir_c-\frac{1}{2}}{ir_c+\frac{1}{2}}. \tag{5.6}$$

The total energy is

$$\begin{aligned} E_{total} &= -\left[\frac{d}{d\nu}\log\widetilde{\Lambda}(\nu)+\frac{d}{d\nu}\log\widetilde{\widetilde{\Lambda}}(\nu)\right]\Big|_{\nu=0}\\ &= \left(\sum_{a=1}^{N_l}\frac{1}{l_a^2+\frac{1}{4}}+\sum_{c=1}^{N_r}\frac{1}{r_c^2+\frac{1}{4}}\right). \end{aligned} \tag{5.7}$$

From the total energy, we can get the eigenvalues of the anomalous dimension matrix of the operators in the scalar sector eq. (3.1) in the β-deformed ABJM theory.

6 The non-supersymmetric three-parameter γ-deformation of ABJ(M) theory

The three-parameter deformation can be performed by replacing all of the ordinary product fg of two fields f and g in the Lagrangian by the following star product [74]:

$$f*g=e^{i\pi\gamma_i Q_j^f Q_k^g\epsilon^{ijk}}fg\,, \tag{6.1}$$

Table 5. $U(1)^3$ charges of the scalars and fermions of the ABJM theory used for γ-deformation.

	Y^1	Y^2	Y^3	Y^4	$\Psi^{\dagger 1}$	$\Psi^{\dagger 2}$	$\Psi^{\dagger 3}$	$\Psi^{\dagger 4}$
$U(1)_1$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	0
$U(1)_2$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
$U(1)_3$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$

where $Q_i^f, i = 1, 2, 3$ are three global $U(1)$ charges carrying by the field f and γ_i's are three real deformation parameters. We choose the $U(1)_i$ charges for the scalars and the fermions as in table 5. The gauge field is neutral under these symmetries. One can see that this deformation degenerates to the one of β-deformation by setting deformation parameters $\gamma_1 = \gamma_2 = 0, \gamma_3 = \gamma$.

After analogous calculation as in section 3, one can find that in the scalar sector and at the two-loop level, the Hamiltonian is still given by eq. (3.10). The differences between β- and γ-deformations may appear at higher loop orders or in other sectors of the theories.

7 Conclusion and Discussions

In this note, we start a study on the integrable structure of β- and γ-deformed ABJ(M) theory, beginning with the scalar sector at the two-loop level in the planar limit. We first perform perturbative computations of the anomalous dimension matrix and express the result as a Hamiltonian acting on alternating $SU(4)$ spin chain. We find that only one term in the Hamiltonian is deformed and that the differences between β-deformation and γ-deformation are invisible in this sector at two loop level. As the undeformed case, the difference between Hamiltonian for deformed ABJM theory and deformed ABJ theory only appears in the prefactor. So in this sector and at this order of the perturbation theory, the violation of parity invariance in the deformed ABJ theory does not affect the integrability. Based on the structure of the deformations, we choose a suitable deformation of the R-matrices. Then the Bethe ansatz equations are obtained through diagonalizing the transfer matrices.

There are several directions worth pursuing. The study here in the field theory side can be extended to full sector and/or higher loop order as in the undeformed case [79]-[85]. It is also interesting to reproduce the Bethe ansatz equation starting from the S-matrix of the spin chain based on the studies in [18, 64, 65, 72]. In the string theory side, one could try to construct the Lax pair and the infinite number of conserved currents on the worldsheet. Even for the undeformed case, the story of IIA string on $AdS_4 \times CP^3$ has already been much richer than the one of IIB string on $AdS_5 \times S^5$, partly because that now the $OSp(6|4)/U(3) \times SO(1,3)$ coset action can only describe a subset of the complete Green-Schwarz action [21–23].

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A Hamiltonian from deformed R-matrices

In this appendix, we compute the Hamiltonian of deformed spin chain in eq. (4.29). For this, we should compute $\widetilde{\tau}'(0,a)$, where the prime denotes the derivative with respect to spectrum parameter u.

$$\begin{aligned}\widetilde{\tau}'(0,a) &= 2^{-L}\,\mathrm{Tr}\sum_i \widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}_{01}(0)\ldots\frac{d\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}_{0(2i-1)}(u)}{du}\Big|_{u=0}\ldots\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}_{0(2L)}(a)\\ &+ 2^{-L}\,\mathrm{Tr}\sum_i \widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}_{01}(0)\ldots\frac{d\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}_{0(2i)}(u+a)}{du}\Big|_{u=0}\ldots\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}_{0(2L)}(a).\end{aligned}\tag{A.1}$$

Where the spectrum parameter u has been set as vanishing. The i-th term in first part of eq. (A.1) can be written down as following

$$\begin{aligned}&2^{-L}\left(\mathbb{P}_{01}\right)^{K_1I_1}_{K_0J_1}\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}_{02}(a)^{K_2I_2}_{K_1J_2}\ldots\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}_{0(2i-2)}(a)^{K_{2i-2}I_{2i-2}}_{K_{2i-3}J_{2i-2}}\delta^{K_{2i-1}}_{K_{2i-2}}e^{i\pi\gamma Q^{K_{2i-1}}\times Q^{I_{2i-1}}}\\ &\delta^{I_{2i-1}}_{J_{2i-1}}\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}_{0(2i)}(a)^{K_{2i}I_{2i}}_{K_{2i-1}J_{2i}}\cdots\left(\mathbb{P}_{0(2L-1)}\right)^{K_{2L-1}I_{2L-1}}_{K_{2L-2}J_{2L-1}}\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}_{0(2L)}(a)^{K_0I_{2L}}_{K_{2L-1}J_{2L}}\\ =\ &2^{-L}\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}_{02}(a)^{I_3I_2}_{J_1J_2}\ldots\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}_{0(2i-2)}(a)^{K_{2i-1}I_{2i-2}}_{J_{2i-3}J_{2i-2}}e^{i\pi\gamma Q^{K_{2i-1}}\times Q^{I_{2i-1}}}\delta^{I_{2i-1}}_{J_{2i-1}}\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}_{0(2i)}(a)^{I_{2i+1}I_{2i}}_{K_{2i-1}J_{2i}}\\ &\ldots\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}_{0(2L)}(a)^{I_1I_{2L}}_{J_{2L-1}J_{2L}}.\end{aligned}\tag{A.2}$$

The i-th term in the second part of eq. (A.1) is

$$\begin{aligned}&2^{-L}\left(\mathbb{P}_{01}\right)^{K_1I_1}_{K_0J_1}\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}_{02}(a)^{K_2I_2}_{K_1J_2}\ldots\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}_{0(2i-2)}(a)^{K_{2i-2}I_{2i-2}}_{K_{2i-3}J_{2i-2}}\left(\mathbb{P}_{0(2i-1)}\right)^{K_{2i-1}I_{2i-1}}_{K_{2i-2}J_{2i-1}}\left(-\mathbb{I}\right)^{K_{2i}I_{2i}}_{K_{2i-1}J_{2i}}\\ &e^{-i\pi\gamma Q^{K_{2i}}\times Q^{I_{2i}}}\left(\mathbb{P}_{0(2i+1)}\right)^{K_{2i+1}I_{2i+1}}_{K_{2i}J_{2i+1}}\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}_{0(2i+2)}(a)^{K_{2i+2}I_{2i+2}}_{K_{2i+1}J_{2i+2}}\ldots\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}_{02L}(a)^{K_0I_{2L}}_{K_{2L-1}J_{2L}}\\ =\ &-2^{-L}\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}_{02}(a)^{I_3I_2}_{J_1J_2}\ldots\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}_{0(2i-2)}(a)^{I_{2i-1}I_{2i-2}}_{J_{2i-3}J_{2i-2}}\delta^{I_{2i}}_{J_{2i}}\delta^{I_{2i+1}}_{J_{2i-1}}e^{-i\pi\gamma Q^{I_{2i+1}}\times Q^{I_{2i}}}\\ &\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}_{0(2i+2)}(a)^{I_{2i+3}I_{2i+2}}_{J_{2i+1}J_{2i+2}}\ldots\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}_{0(2L)}(a)^{I_1I_{2L}}_{J_{2L-1}J_{2L}}.\end{aligned}\tag{A.3}$$

The deformed $\widetilde{\tau}^{-1}(0,a)$ is

$$\widetilde{\tau}^{-1}(0,a) = 2^{L}\left[\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}^{-1}_{0(2L)}(a)\right]^{J_{2L-1}J_{2L}}_{I_1I_{2L}}\ldots\left[\widetilde{\mathfrak{R}^{\mathbf{4}\bar{\mathbf{4}}}}^{-1}_{0(2)}(a)\right]^{J_1J_2}_{I_3I_2},\tag{A.4}$$

where

$$\left[\widetilde{\mathfrak{R}^{4\bar{4}}}^{-1}_{0(2i)}(a)\right]^{J_{2i-1}J_{2i}}_{I_{2i+1}I_{2i}} = -\left(\frac{1}{a+2}\mathbb{I}^{J_{2i-1}J_{2i}}_{I_{2i+1}I_{2i}} e^{i\pi\gamma Q^{J_{2i-1}}\times Q^{J_{2i}}} + \frac{1}{a^2-4}\mathbb{K}^{J_{2i-1}J_{2i}}_{I_{2i+1}I_{2i}}\right). \quad \text{(A.5)}$$

One can use above formula to obtain that

$$\begin{aligned}
&\widetilde{\tau}^{-1}(u,a)\widetilde{\tau}'(u,a)|_{u=0} \\
&= \sum_{i=1}^{L} \mathbb{I}^{K_1\cdots K_{2i-4}}_{J_1\cdots J_{2i-4}} \otimes \left[\widetilde{\mathfrak{R}^{4\bar{4}}}^{-1}_{0(2i)}(a)\right]^{K_{2i-1}K_{2i}}_{I_{2i+1}I_{2i}} \left[\widetilde{\mathfrak{R}^{4\bar{4}}}^{-1}_{0(2i-2)}(a)\right]^{K_{2i-3}K_{2i-2}}_{I_{2i-1}I_{2i-2}} \\
&\quad e^{i\pi\gamma Q^{\tilde{K}_{2i-1}}\times Q^{I_{2i-1}}} \delta^{I_{2i-1}}_{J_{2i-1}} \widetilde{\mathfrak{R}^{4\bar{4}}}_{0(2i-2)}(a)^{\tilde{K}_{2i-1}I_{2i-2}}_{J_{2i-3}J_{2i-2}} \widetilde{\mathfrak{R}^{4\bar{4}}}_{0(2i)}(a)^{I_{2i+1}I_{2i}}_{\tilde{K}_{2i-1}J_{2i}} \otimes \mathbb{I}^{K_{2i+1}\cdots K_{2L}}_{J_{2i+1}\cdots J_{2L}} \\
&+ \sum_{i=1}^{L} \mathbb{I}^{K_1\cdots K_{2i-2}}_{J_1\cdots J_{2i-2}} \otimes \left[\widetilde{\mathfrak{R}^{4\bar{4}}}^{-1}_{0(2i)}(a)\right]^{K_{2i-1}K_{2i}}_{I_{2i+1}I_{2i}} \left(-\delta^{I_{2i+1}}_{J_{2i-1}}\delta^{I_{2i}}_{J_{2i}}\right) e^{-i\pi\gamma Q^{I_{2i+1}}\times Q^{I_{2i}}} \otimes \mathbb{I}^{K_{2i+1}\cdots K_{2L}}_{J_{2i+1}\cdots J_{2L}} \\
&= \sum_{i=1}^{L} \mathbb{I}^{K_1\cdots K_{2i-4}}_{J_1\cdots J_{2i-4}} \otimes \left[\widetilde{\mathfrak{R}^{4\bar{4}}}^{-1}_{0(2i)}(a)\right]^{K_{2i-1}K_{2i}}_{I_{2i+1}I_{2i}} \left[\widetilde{\mathfrak{R}^{4\bar{4}}}^{-1}_{0(2i-2)}(a)\right]^{K_{2i-3}K_{2i-2}}_{J_{2i-1}I_{2i-2}} e^{i\pi\gamma Q^{\tilde{K}_{2i-1}}\times Q^{J_{2i-1}}} \\
&\quad \widetilde{\mathfrak{R}^{4\bar{4}}}_{0(2i-2)}(a)^{\tilde{K}_{2i-1}I_{2i-2}}_{J_{2i-3}J_{2i-2}} \widetilde{\mathfrak{R}^{4\bar{4}}}_{0(2i)}(a)^{I_{2i+1}I_{2i}}_{\tilde{K}_{2i-1}J_{2i}} \otimes \mathbb{I}^{K_{2i+1}\cdots K_{2L}}_{J_{2i+1}\cdots J_{2L}} \\
&- \sum_{i=1}^{L} \mathbb{I}^{K_1\cdots K_{2i-2}}_{J_1\cdots J_{2i-2}} \otimes \left[\widetilde{\mathfrak{R}^{4\bar{4}}}^{-1}_{0(2i)}(a)\right]^{K_{2i-1}K_{2i}}_{J_{2i-1}J_{2i}} e^{-i\pi\gamma Q^{J_{2i-1}}\times Q^{J_{2i}}} \otimes \mathbb{I}^{K_{2i+1}\cdots K_{2L}}_{J_{2i+1}\cdots J_{2L}} \\
&= \sum_{i=1}^{L} \frac{1}{a^2-4}((a-2)\mathbb{I} + (a^2-4)\widetilde{\mathbb{P}}_{2i-1,2i+1} - (a-2)\mathbb{P}_{2i-1,2i+1}\mathbb{K}_{2i-1,2i} \\
&+ (a+2)\mathbb{P}_{2i-1,2i+1}\mathbb{K}_{2i,2i+1}) \qquad \text{(A.6)}
\end{aligned}$$

Similarly, one can get:

$$\begin{aligned}
&\widetilde{\widetilde{\tau}}^{-1}(u,a)\widetilde{\widetilde{\tau}}'(u,a)|_{u=0} \\
&= \sum_{i=1}^{L} \frac{1}{a^2-4}(-(a+2)\mathbb{I} + (a^2-4)\widetilde{\mathbb{P}}_{2i,2i+2} + (a+2)\mathbb{P}_{2i,2i+2}\mathbb{K}_{2i,2i+1} \\
&- (a-2)\mathbb{P}_{2i,2i+2}\mathbb{K}_{2i+1,2i+2}), \qquad \text{(A.7)}
\end{aligned}$$

where we have use the fact that a is purely imaginary. From these two equations, we can get the Hamiltonian given in the main text.

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